Inverse Transform Sampling

Yutao Chen

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The simplest method for sampling from univariate distributions is the *inverse transform sampling*.

Definition 1 (Probability Integral Transform)

Let x be a continuous r.v. with the cumulative distribution function $F_x : \mathbb{R} \to [0, 1]$. We have

$$u = F_x(x) \sim \mathrm{Uniform}(0, 1).$$

Proof. Let F_u be the cumulative distribution function (CDF) of u. We can show that

$$\begin{aligned} F_u(U) &= \Pr\{u \leq U\} = \Pr\{F_x(x) \leq U\} \\ &= \Pr\{x \leq F_x^{-1}(U)\} = F_x(F_x^{-1}(U)) = U. \end{aligned}$$

Note that $F_u(U) = U$ is simply the CDF of a uniform distribution on $[0, 1]$.[1] ∎

Therefore, we can sample from any univariate probability density $p(x)$, of which we can evaluate the inverse CDF F_x^{-1}

$$\begin{aligned} u &\sim \mathrm{Uniform}(0, 1), \\ x &= F_x^{-1}(u) \sim p(x), \end{aligned}$$

by applying the inverse of the probability integral transform.

[1] See also the change-of-variables formula, as discussed in [[Normalizing Flow]].